UNIVERSIDAD AUTÓNOMA DE NUEVO LEÓN
FACULTAD DE INGENIERÍA MECÁNICA Y ELÉCTRICA

“MODAL ANALYSIS OF A PARALLEL KINEMATIC MACHINE: FREEHEX”

POR

JOSUE ISRAEL CAMACHO ARREGUIN

EN OPCIÓN AL GRADO DE
MAESTRÍA EN INGENIERÍA AERONÁUTICA CON ORIENTACIÓN EN ESTRUCTURAS

MARZO, 2017

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SUBDIRECCIÓN DE ESTUDIOS DE POSGRADO

Los miembros del Comité de Tesis recomendamos que la Tesis "Modal Analysis of a Parallel Kinematic Machine: FreeHex" realizada por el alumno "Josue Israel Camacho Arreguin", con número de matrícula 1707090, sea aceptada para su defensa como opción al grado de "Maestría en Ingeniería Aeronáutica con Orientación en Estructuras"

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To my family...

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Abstract

Parallel Kinematic Machines (PKM) are mechanical systems utilized to produce precise operations at high speeds compared to serial machines. Its applications cover a wide range of operations in the automotive and aerospace industry. These machines can be affected by oscillatory effects such as resonant frequencies, which can produce adverse effects on the performance and the useful life of the systems. Therefore the determination of these characteristics is an important element in the design and operation of these machines.

The research developed in this thesis addresses the identification of these resonant frequencies in the possible working volumes of a PKM called FreeHex. This machine is used to perform high speed machining operations and in situ rapair of aerospace equipment. Two methods are developed and described to estimate the resonant frequencies.The first is formulated through the Finite Element Method (FEM), and given the initial configuration of the FreeHex is capable to estimate the resonant frequencies within the working volume of that configuration. The second method is intended to validate the model described by the FEM. As the FreeHex is able to achieve a several configurations the analysis is performed to a single one and then the experimental and the finite element analysis model are compared. The experimental results are obtained by impact testing trough Multiple Input Single Output (MISO) method on the structure. The average error between the FEA models and experimental results lies in a value of 3.94 % showing a good correlation between both methods and making possible to implement the algorithms here developed.

Chapter 1

Introduction

1.1 State of the Art

Nowadays, there are a wide variety of technologies being applied to different fields of knowledge, one of which is focused on the development of instruments, equipment and systems for aerospace applications. One of these novel instruments has been developed at the University of Nottingham and it is known as the FreeHex. This machine is a Parallel Kinematic Machine (PKM) for *in situ* repair of aerospace equipment. These PKM are robots that perform motions and rotations by three or more actuators connected in parallel. PKM have attracted attention as machine tools due to their conceptual potentials in high precision and rigidity because of their closed kinematics loops.

Parallel Kinematic Machines offer advantages for certain operations with respect to serial machines. PKMs experience less limitations related to its acceleration for machining and those are able to achieve high speeds and high precision operations. However, certain conditions may affect PKMs, especially by resonances and displacement of the actuators [1] . As resonances are dependant of stiffness (particularly on the X and Y axes [2]) and stiffness is directly related to the configurations of the PKM. For this reason the prediction of the natural frequencies becomes more complicated. This complication also relies on the fact that many configurations must be analysed to properly avoid these undesired effects when performing machining operations.

Several studies have been developed for PKM and the modal analysis of these machines. Most of these studies consider an arbitrary configuration for specific modes of vibration and some suggest changes in the design in order to modify such frequencies for very specific arrangements [2]. Other models intend to analyse PKMs in a relative not complex configuration in order to be able to solve the dynamical behaviour. These show to be convenient as most PKM are capable to operate in a fixed work volume. But for the FreeHex shows to be inconvenient as

the work volume varies with respect to the configuration of the pods.
Another approach is achieved by Xiaohui Han *et al* as a PKM is analysed by a super model in a FEA software [18]. T. Huang *et al* describe this method as one that requires a considerable amount of time as the process involves the continuous modification of the model, the meshing and the connections between the different components of the PKM [19]. It is also easy to understand that a super model such as the one proposed by [18] applied to the FreeHex would require long periods of time and computational resources to obtain the solutions for the different working volumes. These last sentence portraits a model impossible to characterise the resonances in a practical application. However, some problems related to stiffness characterisation of these machines remain unsolved. The improvement of such analysis would produce a better level of coincidence between theoretical and practical results. Considering that the dynamical behaviour of a PKM implies phenomena that cannot be easily modelled such as mechanical joints (stiffness, coulomb damping and tolerances [3]). These characteristics impose noise and vibrations that may lead to instabilities for some operations. Although some models consider mechanical joints [6] a simplified model and applied to PKMs would be useful. Zou et al [5] studied a PKM and modelled the actuators as rigid elements and the joints as virtual springs. Long [4] studied the dynamical behaviour of a Gough-Stewart machine considering a flexible platform but with rigid actuators.

Several mathematical models have been developed in order to describe the kinematics for the FreeHex and as a consequence these models allow to predict the volumes of work for the machine under different configurations[16]. This possibility to operate in many working volumes represents one of the several advantages with respect to other PKMs developed by other universities and companies. Various tests have shown the advantages and capabilities of the FreeHex especially as equipment for the aerospace industry. As the requirements of the aerospace industry usually imply complex geometries, special machining operations and complex locations where the machining must be performed. Some abnormalities have also been detected. These abnormalities allow the improvement of the performance in the FreeHex. Differences between machining and designing have been related to the displacement of the bases of the actuators during some machining operations. These phenomena is probably related to oscillatory effects.

1.2 Hypothesis

Several studies are using very detailed models, with respect to the Finite Element Method, applied to the analysis of complex machines used for additive manufacturing, milling and cutting processes. Usually these studies are related to elements requiring a deep study of structural behaviour over its body, boundary conditions and loads that are difficultly represented by analytical methods. These studies have proven to be precise but its own extension in elements and connecting conditions have limited them to be applied only on fixed configurations.

Based on the theory of the Finite Element Method (FEM) this project expects to develop staggering efficient set of algorithms to be implemented and performed a milestone analysis through commercial software. These algorithms will be capable to predict the different resonant frequencies of the FreeHex in the many, configurations in order to operate out of unstable conditions. These last sentence implies a short amount of time of analysis with a high level of accuracy overcoming the actual studies that imply a significant amount of time to acquire results.

1.3 Objectives

As high speed and precision machining operations are leading engineers and researchers to the application of parallel kinematic machines in aerospace and automotive industry, it is predominant to understand the behaviour of such machines in the different operations of the process. The dynamics of machining needs to be addressed such as cutting forces, vibrations and smoothness of the cutting motion [27]. Considering the advantages that the FreeHex represents for the technology of machining processes, the next series of objectives are proposed in order to address the vibrational behaviour:

- To develop a Finite Element Analysis Model capable to accurately represent the dynamic characteristics of the FreeHex with an optimal performance. Being mainly the resonances so these conditions are avoided.
- This model shall be able to provide enough information so unstable conditions of operation for the FreeHex can be predicted and avoided in a set of the different configurations achieved by the FreeHex. The information acquired trough these models can be used by the teams responsible for the control of the machine to avoid certain cutting speeds.
- To achieve a good structural representation of the system. Such representation shall be validated by the comparison between the characteristics described by the proposed model with respect to the ones obtained through

experimentation on the FreeHex and there for expanded the use of the model to the many configurations achieved by the FreeHex.

1.4 Methodology

It is possible to understand that the complication of the analysis for a PKM relies on the stiffness matrix, which is associated to the configuration of the FreeHex, and the complication related to the modelling of the joints in the machine. Being inconvenient to represent them as infinite stiff elements. This allows to understand that a very detailed model may result inconvenient for the different arrangements that the FreeHex may achieve. Given the complex geometries involved and the computational resources required for the analysis by a super model. One possible solution is the reduction from a super model to a simplified one of equivalent properties.

The FreeHex poses identical actuators and a complete analysis of one of them and the joints conforming it could be performed in order to acquire properties such as stiffness, damping and mass distribution. Later these results could be included in the simplified model in order to incorporate result from both models. For this analysis it is proposed the combination of two commercial programs (MatLab and ANSYS) where the intention is to take advantage of the capabilities of both. One where the algorithm will be created and the second will solve such algorithm. Finally the results obtained by these models will be compared with the data acquired by experimentation on the FreeHex. These last results are planned to be obtained by tap testing trough Multiple Input Single Output (MISO) on the structure.

As the FreeHex is able to achieve a several configurations it is intended to perform the analysis to a single one and compare the experimental and the finite element analysis model. Once validated the FEA models could be applied in the acquisition of the resonances for future operations. Working out of instabilities that can lead to undesired perturbations of the system and therefore achieving a better performance of the FreeHex. In order to simplify the already described process the next diagram is presented as a summary of the methodology to be follow.

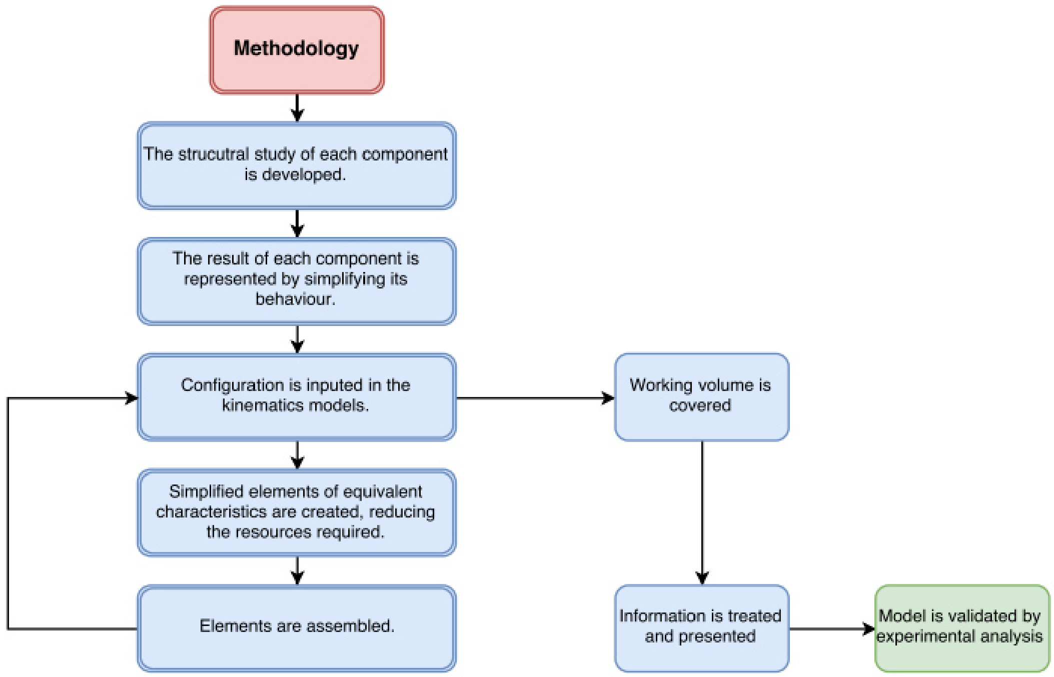

Figure 1.1. Diagram of the methodology proposed to acquired the resonant frequencies for the FreeHex by the use of the FEA.

1.5 Scope and Limitations

The current project aims to acquire the resonant frequencies for the FreeHex in its many possible configurations. In order to acquire these frequencies, the user must provide as input the configuration of the pods. In order to validate the FEA models, these results must be compared with those acquired by experimentation. As previously stated, the FreeHex can adopt many configurations and this project is considering to evaluate both set of results (FEA and experimental) to a single configuration. This comparison is no considering any force or excitation generated during any machining process and does not consider the future possible objects where it will work.

1.6 Summary

The present project is divided into five chapters. Each chapter is focused on preparing to understand the next section. As a brief summary the next list is presented.

1. **Chapter 1** The needs, requirements and expectations of this project are presented. Several models are cited so those can be compared to the proposed method and the possible advantages it represents.

2. **Chapter 2** The theoretical bases of vibrations and finite element method required to understand further methodology are explained.

3. **Chapter 3** The methodology to obtain the structural characteristics of the FreeHex is developed. Presenting it from the study of each component and later assembling these analysis in one called global.

4. **Chapter 4** The experimental process to acquire the oscillatory characteristics of the FreeHex is explained. From configuration of the sensors used to the post processing of the signals acquired, the analysis is documented and presented.

5. **Chapter 5** The final results obtained in chapters 3 and 4 are compared. From these comparisons the conclusions are established.

Chapter 2

Theoretical Bases

This chapter gives the theoretical fundamentals that represent the basis for the procedures established in the next sections.In here, it is presented a brief introduction to the mathematical tools used along with the dynamical analysis, for both theoretical and practical testing.

2.1 Introduction

2.1.1 Vibrations and Modal Analysis

Vibrations are changes in the configuration of a system with respect to time, referred to an equilibrium state. These changes are related to the transformation of the energy in the system as potential and kinetic energy and the dissipation of it (damping) [7].

Excitation may come from different sources such as the movement of the structure where the system may be based on fluids in movement or even internal loads such as engines or random events. These excitations may be classified by their behaviour as periodic (harmonic simple and complex), non-periodic (transient and impulsive) and random (stationary and non-stationary). In order to perform the analysis of a given system, it must be considered that the response depends on the shape and place of application as well as the characteristics of the structure such as mass and stiffness distributions in addition to energy dissipation characteristics.

Almost any structure can be brought to resonate, which means that the given system will vibrate in an excessive motion (even by a small excitation force) and is caused mainly by the interaction between inertial and elastic properties of the structure [8]. This means that the system is resonating and the amplitude of the stress, deflection and strain will exceed the caused by a static load of the same value.

The importance of the development and analysis of models for mechanical systems relies on the ability to predict the dynamical behaviour of the structure under analysis such as resonant frequencies, modes and damping factors allowing those systems to operate at stable configurations achieving longer life cycles and better operational performance. These characteristics are obtained by a process called Modal Analysis and relies on the fact that the vibration response of a linear time-invariant dynamic system can be expressed as the linear combination of a set of harmonic motions called the natural modes of vibration. This concept is a milestone in order to analyse a complex wave form as the combination of a harmonic signals through the Fourier transformation [10].

2.2 Theoretical Modal Analysis

Modal analysis involves both the theoretical and experimental techniques. It contains a physical model system as a description of mass, stiffness and energy dissipation properties. A realistic model will involve these qualities in terms of spatial distribution as a set of equations, which usually are represented as the mass, stiffness and damping matrices. The superposition principle of linear dynamic systems allows to perform the transformation of these equations into a classical eigenvalue and eigenvector problem. When the mechanical system under study implies complex conditions related to material properties, load distribution and complex boundary conditions, an equivalent system with simplified conditions may result in a good approximation with benefits related to cost/precision ratio[11].

2.2.1 Motion Equations

It is required to stablish the motion equations of a given system in order to obtain the natural frequencies and modes as stated by Inman [23]. Several mathematical tools have been developed for such purpose such as the second law of motion of Newton-Euler or the Hamilton's equations.

In the next section a brief explanation on how the motion equations for a single degree of freedom and later a multiple degree of freedom can be acquired. The purpose to develop this analysis is to present a better perspective of the topics treated in this text.

Mass-Spring System

Suppose the simplest possible system, the spring-mass system represented by the Figure 2.1, where the mass m is displaced a distance X_0 and then the mass is quickly released. Equation (2.1) considers the mass to be restricted to displace only in a horizontally movement and without dissipation of energy.

$$m\ddot{X} + kX = 0. \tag{2.1}$$

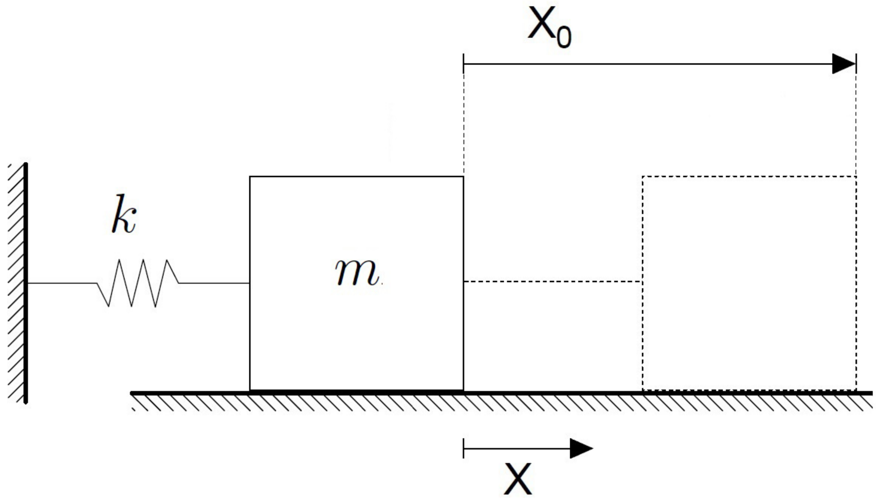

Figure 2.1. Spring-Mass system.

Where k corresponds to the stiffness of the spring in the system, X is the position of the mass m, and $\ddot{X}$ corresponds to the acceleration of the mass. When solving the Equation (2.1), it is possible to conclude that the mass m will oscillate in a simple harmonic movement given by the Equation (2.2).

$$X = X_0 \sin(\omega t + \epsilon). \tag{2.2}$$

Being dependent of the time t, where $\omega^2 = k/m$ and ϵ is the phase angle. The period and frequency rely on the basic characteristics of the system, such as spring stiffness and the magnitude of the mass. The amplitude of the oscillations can be modified by altering the initial elongation of the spring but the frequency will remain constant. This frequency is the so called natural frequency and the harmonic movement of the mass is the normal mode of vibration [24].

Spring-Mass System with n Degrees of Freedom

Consider the system shown in the Figure 2.2, with n masses and n springs. Restricting the movement of the springs but in the axial direction the system is declared to be of n degrees of freedom. Then if the system is brought to oscillate in a harmonic motion it will vibrate in n different mode shapes. The amplitudes of displacement of the system and the frequencies of the motion shall take different values for each of the n modes. These sets of displacements and frequencies are called proper modes and resonance frequencies.

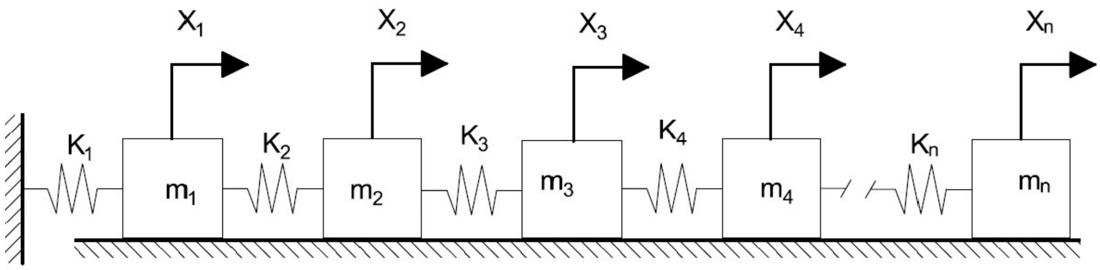

Figure 2.2. Spring-mass system with n degrees of freedom.

The set of Equations (2.3) describes the movement of the system shown in the Figure 2.2 and the solution of this set of equations implies the resolution of n equations of second order [13].

$$\begin{aligned} m_1\ddot{X}_1 + X_1(k_1+k_2) - X_2(k_2) &= 0, \\ m_2\ddot{X}_2 - X_1(k_2) + X_2(k_2+k_3) - X_3(k_3) &= 0, \\ m_3\ddot{X}_3 - X_2(k_3) + X_3(k_3+k_4) - X_4(k_4) &= 0, \\ m_4\ddot{X}_4 - X_3(k_4) + X_4(k_4+k_5) - X_5(k_5) &= 0, \end{aligned} \tag{2.3}$$

$$\vdots$$

$$m_n\ddot{X}_n - X_{n-1}(k_n) + X_n(k_n+k_{n+1}) - X_{n+1}(k_{n+1}) = 0. \tag{2.4}$$

The motion equation for the nth mass is express by Equation (2.5)

$$m_i\ddot{X}_i - X_{i-1}(k_i) + X_i(k_i+k_{i+1}) - X_{i+1}(k_{i+1}) = 0. \tag{2.5}$$

The system of Equations (2.3) is represented as a matrix by Equation (2.6).

$$[M]\left\{\ddot{X}\right\} + [K]\left\{X\right\} = 0, \tag{2.6}$$

Where:

$[K]$ is the sttiffness matrix of the system.

$[M]$ is the mass matrix of the system.

$\left\{\ddot{X}\right\}$ is the acceleration vector of the system.

$\{X\}$ is the vector position of the system.

The stiffness matrix for a given system is represented by Equation (2.7).

$$[K] = \begin{bmatrix} k_{1,1} & k_{1,2} & k_{1,3} & k_{1,4} & ... & k_{1,n} \\ k_{2,1} & k_{2,2} & k_{2,3} & k_{2,4} & ... & k_{2,n} \\ k_{3,1} & k_{3,2} & k_{3,3} & k_{3,4} & ... & k_{3,n} \\ k_{4,1} & k_{4,2} & k_{4,3} & k_{4,4} & ... & k_{4,n} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ k_{n,1} & k_{n,2} & k_{n,3} & k_{n,4} & ... & k_{n,n} \end{bmatrix}. \tag{2.7}$$

And the mass matrix $[M]$ is represented as:

$$[M] = \begin{bmatrix} m_1 & 0 & 0 & 0 & ... & 0 \\ 0 & m_2 & 0 & 0 & ... & 0 \\ 0 & 0 & m_3 & 0 & ... & 0 \\ 0 & 0 & 0 & m_4 & ... & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & ... & m_n \end{bmatrix}. \tag{2.8}$$

2.3 Experimental Modal Analysis

Modal testing is defined as the process of characterising the dynamic behaviour of the structure in terms of the modes of vibration from experimental data, related to the acquisition and measurement of the frequency response function (FRF). The common process is to apply a known excitation at one location of the system and measure the vibration responses in one or several locations [12].

As previously stated for a linear system, the response for a given structure will be a linear superposition of all the excited modes where each of these modes can be represented by a model of one degree of freedom. Under this statement, a mechanical system with n degrees of freedom shall present n peaks (one related to each mode) in the frequency spectrum of its response. These peaks represent the modal frequencies and those are determined by observing the FRF.

2.3.1 Excitation Techniques

The excitation techniques are classified into two groups in function of the structure union during test:

- In contact with the structure during all the duration of the test, it provides a continuous excitation or even transient. The main methods are listed below.

  Sinusoidal excitation. The excitation force contains one frequency at every interval of time. Which varies with a certain step in a given range allowing the structure to responds to one frequency at the time. Single input sinusoidal excitation can be restrictive in time resources despite its superior effectiveness which can be solved by multi input/multi output testing.

  Random excitation. It is a stationary random signal that presents a Gaussian distribution, which contains all frequencies in the range of interest. Usually the signal excites the structure for several periods until the desired mean level and variance characteristics are achieved

  Pseudo random excitation. The signal is generated in the frequency domain being periodic, of constant amplitude and random phase angle. This excitation eliminates the possible leakage problem present in random excitation. It does not work well in the presence of non-linearities, distortions that cannot be removed by averaging.

- In contact with the structure by a differential period of time (impact).

  A commonly used technique is the impact through a hammer for modal analysis. The duration of the impact is very short, compared to the time measure response. This technique allows to obtain modes from 5 to 10 KHz, depending on the hammer type.

  There exist two factors that are fundamental to obtain the correct results when performing test through hammer technique. Where the selection of the tip in the hammer and the choice of an appropriated windowing of the signal measured for the response.

  The first is related to the excitation of the modes of interest in a certain range. If the tip is too hard, the lower frequency modes will not be properly exited and if the tip is too soft, the upper modes will not be exited as well as a poor coherence will be obtained. The second is related to the proper acquisition of the response, for example if the structure under analysis is a mechanical system with very low values of damping, the response measured will not be zero at the of the sampling interval.

2.4 Dissipation of Energy

Imagine a mechanical system with one degree of freedom which is perturbed by force F in a infinitesimal segment of time t. Let the system oscillate freely and dissipate its initial energy until its displacements decreases to zero (Figure 2.3). The response of the system would be given by Equation (2.9).

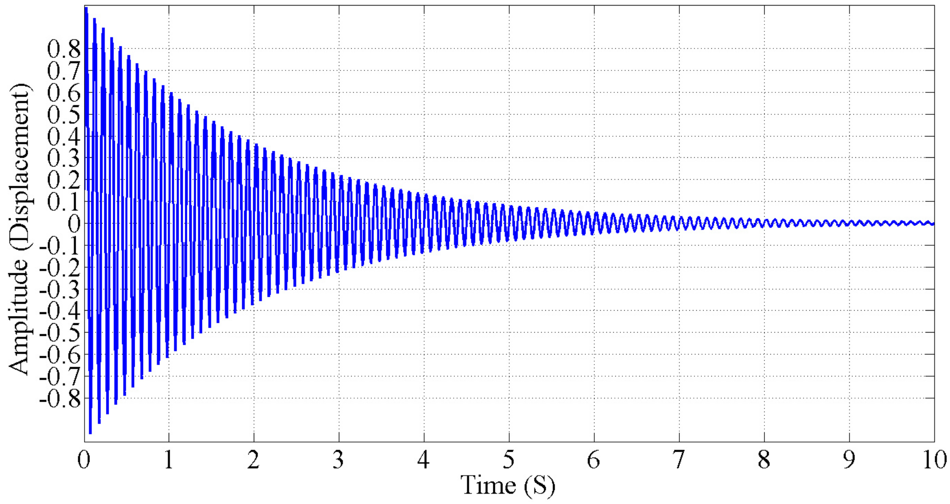

Figure 2.3. Free Response of a mechanical system to a perturbation.

$$X = X_0 e^{-\zeta \omega_n t} cos\,(\omega_d t - \phi_0) \tag{2.9}$$

Let the Equations (2.10) and (2.11) represent two amplitudes of oscillation X_1 and X_2:

$$X_1 = X_0 e^{-\zeta \omega_n t_1} cos\,(\omega_d t_1 - \phi_0) \tag{2.10}$$
$$X_2 = X_0 e^{-\zeta \omega_d t_2} cos\,(\omega_d t_2 - \phi_0) \tag{2.11}$$

And X_1 and X_2 be two consecutive amplitudes acquired one cycle apart. Then the correspondent relation between it sampling time t_1 and t_2 is given by:

$$t_2 = t_1 + (2\pi/\omega_d) \tag{2.12}$$

Then the rate at which the amplitude decays per each cycle can be represented as:

$$\frac{X_1}{X_2} = \frac{X_0 e^{-\zeta \omega_n t_1} cos\left(\omega_d t_1 - \phi_0\right)}{X_0 e^{-\zeta \omega_n (t_1 + 2\pi/\omega_d)} cos\left(\omega_d \left(t_1 + 2\pi/\omega_d\right) - \phi_0\right)} = e^{\zeta \omega_n (2\pi/\omega_d)} \tag{2.13}$$

Finally the logarithmic decrement of the displacement can be represented as:

$$ln\left(\frac{X_1}{X_2}\right) = 2\pi\zeta\frac{\omega_n}{\omega_d} = \frac{2\pi\zeta\omega_n}{\sqrt{1-\zeta^2}\omega_n} = \frac{2\pi\zeta}{\sqrt{1-\zeta^2}} \tag{2.14}$$

This logarithmic decrement represents the rate at which the amplitude of vibration decreases (usually expressed as Δ). Δ represents the dissipation of energy and therefore a method to obtain the damping coefficients for a mechanical system [23]. Where the damping coefficient is represented by ζ. Equation (2.14) can be used to express ζ as:

$$\zeta = \frac{\Delta}{\sqrt{4\pi^2 + \Delta^2}} \tag{2.15}$$

2.5 Fourier Transform

The Fourier transform is one of the most used methods to change data sequences and functions from the time domain to the frequency domain. Its applications go from signal filtering to biomedical applications [28].

2.5.1 The Discrete Fourier Transform

The Discrete Fourier Transform is the equivalent of the continuous Fourier Transform but only being sampled at constant intervals of time. As any signal stored and processed by a PC, it is required to have finite length [29].

The Discrete Fourier Transform is defined as:

$$F\left[j\right] = \sum_{n=0}^{N-1} f\left[k\right] e^{-2\pi i n j/N} \qquad 0 \leq j \leq N-1 \tag{2.16}$$

$$f[n] = \sum_{j=0}^{N-1} F[j]\, e^{2\pi i n j/N} \qquad 0 \leq n \leq N-1 \tag{2.17}$$

The interpretation of the Equations (2.16) and (2.17) is that at point n, the sequence value $f[n]$ is a linear combination of the values of N sinusoids, $e^{0}...e^{(2\pi/N)n(N-1)}$. The coefficients of the sinusoids are $F[0]\,,\,,F[N-1]$, respectively, and their frequencies are j/N cycles per sample or $2\pi j/N$ radians per sample.

2.6 The Process of the Finite Element Method

The finite element method (FEM) is a powerful tool used to model problems that involve conditions that cannot be solved through analytical tools. For this reason FEM has become an essential tool in the design and modelling of different areas of knowledge such as physics, structural analysis, fluid dynamics, electromagnetism and others [21]. These phenomena are present in continuous elements, such as solids, liquids or gases and involve a wide variety of variables which usually are dependant on position and position, that complicates the solution. The main basis for the FEM relies on the fact that a continuous component of infinite elements are reduced to a defined set of finite elements by dividing the domain in a certain number of components and express their behaviour as a series of approximations to each of those elements. Those are defined on specific locations of each elements that commonly are known as nodes. The nodes are usually located on the boundaries of every element that connect adjacent elements. The FEM can be expressed by three main stages:

1. **Preprocessing**
   The system under analysis is created. Usually some environments allow the development of geometries by a Computer Assisted Design (CAD) software. Over this model the mesh is performed, whether it be automatic or fully controlled by the user. The goal in this stage is to bring the geometrical model to a mathematical model correlated to the properties of the system such as material properties, loads acting on the structure and boundary conditions [22].

2. **Analysis**
   The information prepared in the preprocessing environment is implemented in a mathematical language usually as matrices which constructs and solves the system of algebraic equations.

3. **Postprocessing**
   The information obtained through the solution of the system is analysed in order to understand and present the information acquired through graphical methods or values presented in certain locations of interest.

2.7 Summary

The fundamentals of vibration and modal analysis necessary to understand the following chapters have been presented, as the next chapters take as a fact such concepts and methods used.

Topics as the stiffness and mass matrices have been explained, so those concepts can be related to the modelling developed and the relations to the mathematical background behind the algorithms explained. Other topics as the Fourier transform and the general process of FEM have also been introduced as they provide help to understand the response of the systems under analysis. It is strongly recommended to review the references provided if the intention is to acquired deeper explanation of each topic and detailing it would result out of the scope of this text.

Chapter 3

Finite Element Model of the FreeHex

This chapter presents the method developed to analyse the dynamical behaviour of the FreeHex. The first part deals with the individual analysis of its components to finally joint individual results in a system called global which will provide proper modes and frequencies of the resonances.

3.1 Introduction

The proposed solution is to consider accurately the structural behaviour of all the elements conforming the FreeHex. This implies to perform analysis such as stiffness estimation of the most complex parts separately from the complete assembly. When the structural parameters have been properly estimated the result would be included in the complete assembly of the FreeHex. This method allows to simplify from solid elements to shells, beams and link elements meaning the reduction of the number of elements and nodes generated in the FEM model (3.1). This is very convenient as these models intend to evaluate the dynamical behaviour of the FreeHex for working volumes that are related to different configurations. This reduction of elements is connected to hundreds of representations of different configurations of the PKM under analysis and therefore saving resources to acquire the solutions.

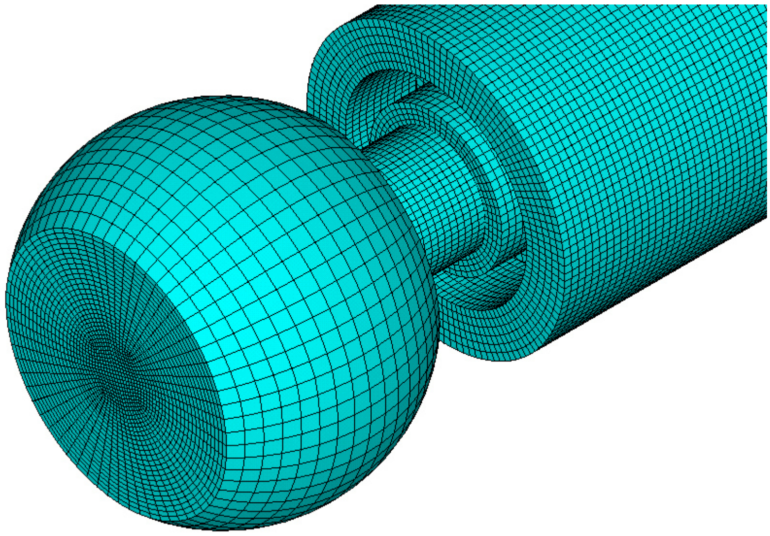

Figure 3.1. Finite element model of one of the linear actuators.

3.2 Structural Analysis of the Main Actuators

Modelling the linear actuators as link elements in a parametrical method would result very convenient as the reduction of elements can be significant. This simplification relies on the fact that the boundary conditions of the actuators impose them to be working as link elements.
Since the position of the moving platform is established by the location of the linear actuators (legs) as well as the length of those elements (Figure 3.2), it is important to evaluate the stiffness in the actuators as function of the operational length. This evaluation permits to include stiffness and mass in the global FEM for the FreeHex. As part of the investigation for each of the linear actuators a series of analysis where developed. Applying a uniform load at one of the ends of the actuator and adjusting the position of the different components in the linear actuator so the length within its operational range (231-321 cm) has been covered. The results of these simulations is a deformation of the actuator in each configuration (Figure 3.3), which are related to an equivalent stiffness. From these analysis the information was processed and displayed in the Table 3.1, represented in the Figures 3.3 - 3.5.

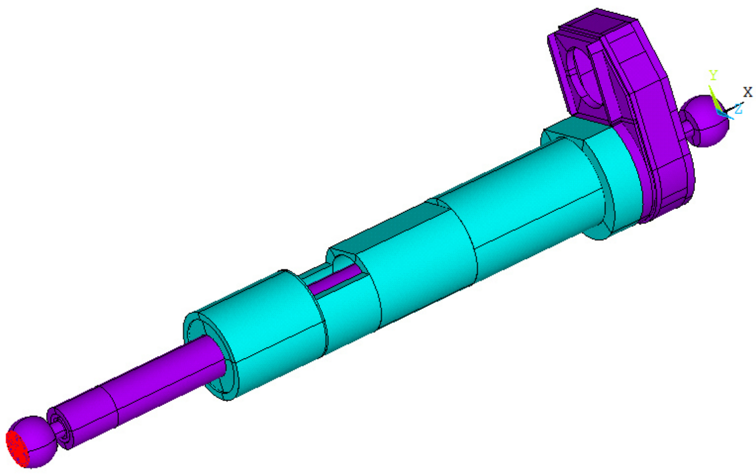

Figure 3.2. Geometry of the assembly of the linear actuator.

Length (m)	Deformation (m)	Stiffness (N/m)
0.231	9.81396E-11	14369326.7
0.241	1.05118E-10	13415461.66
0.251	1.11951E-10	12596627.53
0.261	1.18784E-10	11871958.56
0.271	1.25617E-10	11226172.7
0.281	1.3245E-10	10647021.56
0.291	1.39284E-10	10124657.76
0.301	1.46117E-10	9651153.213
0.311	1.5295E-10	9220009.548
0.321	1.59783E-10	8825703.539

Table 3.1. Results of stiffness and deformation for every 10 cm estimated from the FEA model. This result correspond to the linear actuators in the operational length.

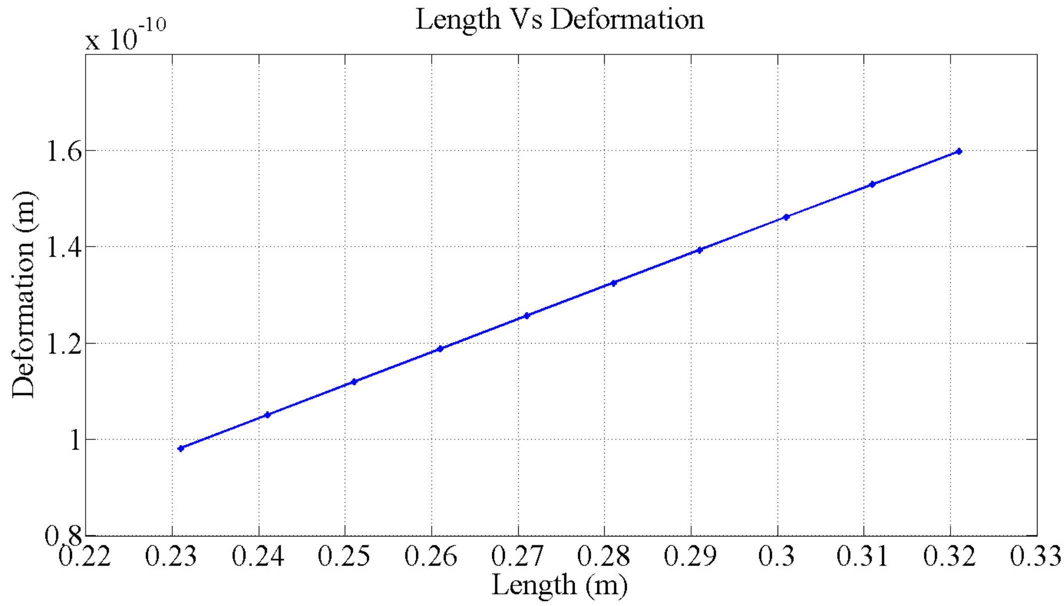

Figure 3.3. Deformation of one of the linear actuators with respect to length.

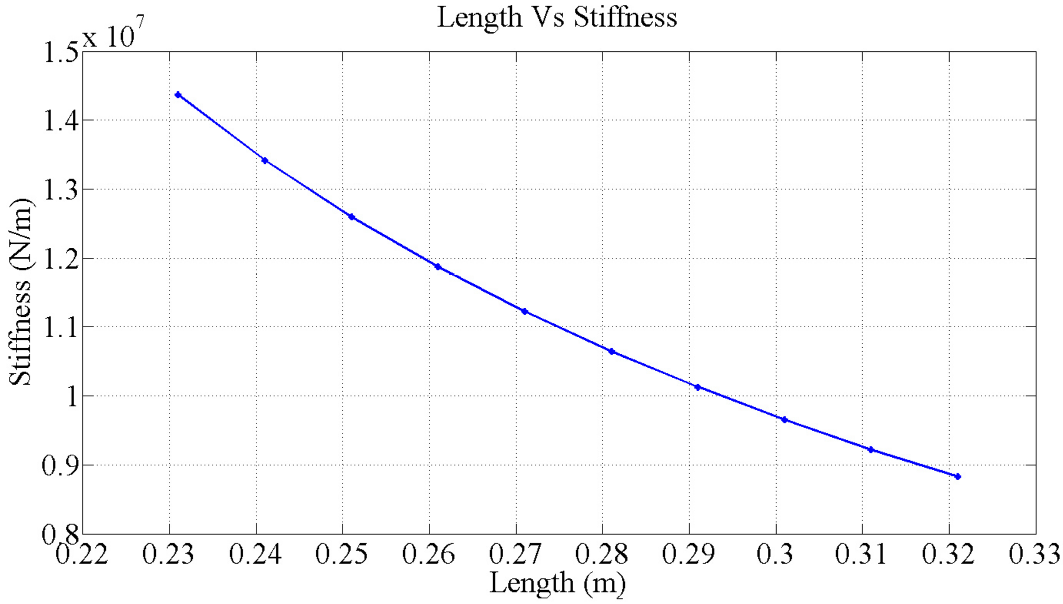

Figure 3.4. Stiffness of one of the linear actuators with respect to length.

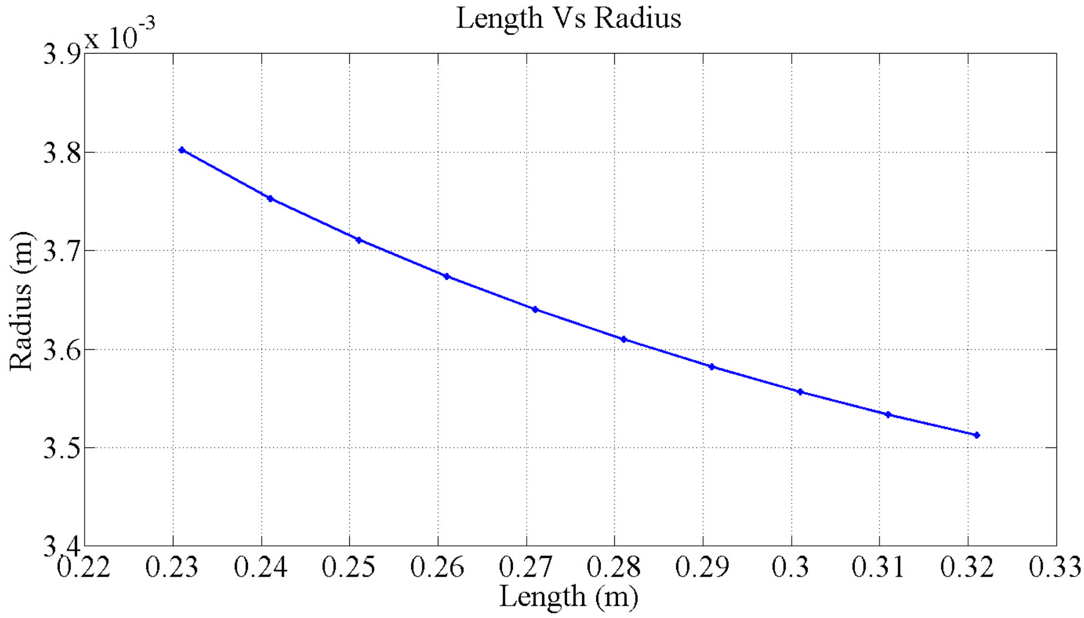

Figure 3.5. Relationship between cross sectional radius and length.

In order to include an accurate and efficient representation of stiffness and mass in the global analysis for the FreeHex a logarithmic regression was developed. This regression allows to obtain the relation between length and cross sectional area. The calculated function permits to evaluate the equivalent values of stiffness for each actuator with an average error of 0.15 %. This function is represented by a logarithmic expression as the minimum stiffness will tend to the stiffness of the weakest element conforming the system. This last simplification proved to be effective in the reduction of time and computational resources required as it is possible to reduce from one million nodes and fifty three contact conditions to 2 nodes containing equivalent properties and therefore maintaining an accurate representation of stiffness and mass for the system. This significant reduction is effectuated as each actuator is imposed, by its boundary conditions, to be acting as an element subject only to compressive and tensile loads. The link element (properly considered) is capable to represent this compressive/tensile behaviour and the mass distribution of the actuator with only two nodes.

3.3 Structural Analysis of Spindle and Spindle Holder

The second series of elements under analysis belong to the spindle and the spindle holder, which are modelled as a series of beams working together.These components are studied experimentally in section §4 and these analyses provide information to feedback the FEA models.

Some of the main conditions for the analysis are considered ideal. These elements are supposed to behave as a series of connected beams with equivalent stiffness. Assuming that the components belonging to the spindle would only be subject to bending modes, for instance the spindle and spindle holder, then it may result possible to compare both to a par of springs working in parallel as shown in Figure 3.6 which is represented by the Equation (3.1).

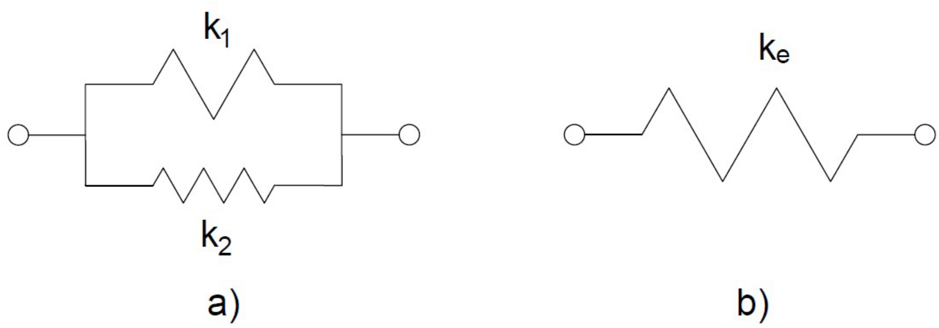

Figure 3.6. a) System of springs in parallel . b) Equivalent stiffness.

$$k_e = k_1 + k_2. \tag{3.1}$$

If Equation (3.1) is now represented by matrices the equivalent stiffness for the mentioned elements would be:

$$[k_{ae}] = [k_p] + [k_s]\,. \tag{3.2}$$

As previously stated, the elements under investigation represent beams and then the proper formulation for the analysis must be taken into account [14]. For a beam element the stiffness is represented as:

$$[k^e] = \frac{EI}{L^3}\begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix}. \tag{3.3}$$

And the inertial matrix would be expressed:

$$[m^e] = \frac{\rho AL}{420}\begin{bmatrix} 156 & 22L & 54 & -13L \\ 22L & 4L^2 & 13L & -3L^2 \\ 54 & 13L & 156 & -22L \\ -13L & -3L^2 & -22L & 4L^2 \end{bmatrix}. \tag{3.4}$$

Where:

E is the elasticity modulus of the material that conforms the element e.

I is the moment of inertia.

L Is the length of the element.

ρ is the density of the section.

A is the transverse area of the element.

These properties can be obtained for the spindle holder from the characteristics of the material and by certain measurement procedures. However, it results more complicated to obtain such properties for the spindle.

In order to represent the dynamical behaviour of the spindle components, some idealisations are established in order to approximate the unknown parameters. Then a set of considerations allow to calculate the unknown features by the establishment of the next idealisations:

- The cross sectional area of the spindle is considered to be formed by a solid transverse section, being a circular area of constant radius.
- The spindle density is considered to be uniform on all the component.

These assumptions lead to simplify the initial problem to a single unknown parameter being E, which is approximated by the information provided by the experimental analysis of the spindle. For these a beam subject to bending modes and freely supported at the ends is considered under analysis. The equation governing the oscillatory behaviour of a beam is given by Equation (3.5):

$$-\frac{\partial V}{\partial x} + P(x,t) = \bar{m}\frac{\partial^2 y}{\partial t^2} \tag{3.5}$$

Where:

- V is the shear force acting over the beam.
- P is the distributed load per unite of length..
- $\bar{m}$ is the mass per unit of length.

From the theory of elasticity mentioned by Hibbeler [25]:

$$V = EI\frac{\partial^3 y}{\partial x^3} \tag{3.6}$$

Substituting Equation (3.6) in Equation (3.5):

$$EI\frac{\partial^4 y}{\partial x^4} + \bar{m}\frac{\partial^2 y}{\partial t^2} = P\left(x, t\right) \tag{3.7}$$

As the spindle is considered to be vibrating freely:

$$EI\frac{\partial^4 y}{\partial x^4} + \bar{m}\frac{\partial^2 y}{\partial t^2} = 0 \tag{3.8}$$

Equation (3.8) can be solved which implies that the solution can be expressed as the product of two functions, one being dependent on the position $G(x)$ and another being dependent of the time $F(t)$:

$$y(x, t) = G(x)F(t) \tag{3.9}$$

Applying Equation (3.9) in Equation (3.8)

$$\frac{EI}{\bar{m}}\frac{G^{IV}(x)}{G(x)} = -\frac{F^{II}(t)}{F(t)} \tag{3.10}$$

As the first term of the Equation (3.10) is only dependant of the position, and the second is only dependent of the time each of the terms must be equal to a constant designated as ω^2, then:

$$\frac{EI}{\bar{m}}\;\frac{G^{IV}(x)}{G(x)} = -\frac{F^{II}(t)}{F(t)} = \omega^2 \tag{3.11}$$

$$G^{IV}(x) = \frac{\bar{m}\omega^2}{EI} G(x) \tag{3.12}$$

Where the next notation results to be very convenient to use:

$$a^4 = \frac{\bar{m}\omega^2}{EI} \tag{3.13}$$

$$a = (aL)^2 \tag{3.14}$$

And finally:

$$\omega = C\sqrt{\frac{EI}{\bar{m}L^4}} \tag{3.15}$$

Where ω represents the natural frequency of the beam, and it will be determined by the boundary conditions of the system. As ω is a solution of n possible equations, n omegas will exist, and then it must be determined the rage of interest for the given solution.

$$\omega_n = C_n\sqrt{\frac{EI}{\bar{m}L^4}} \tag{3.16}$$

The boundary conditions for a free beam (both ends free) are:

$$M(0,t) = 0 \quad or \quad G^{II} = 0 \tag{3.17}$$

$$V(0,t) = 0 \quad or \quad G^{III} = 0 \tag{3.18}$$

When these conditions are applied on the particular solution for a free beam as shown by Paz [15], the proper values for the system are obtained and expressed in the Table 3.2. Later the natural frequencies for the system can be calculated when these constants are applied to Equation (3.16).

n	C_n
1	22.3733
2	61.6728
3	120.9034
4	199.8594
5	298.5555

Table 3.2. Values for the constant C_n for a free beam.

An approximation to the equivalent modulus of elasticity for the spindle and spindle holder can be estimated by the use of the experimental data acquired and explained in Section §4 (correspondent to the fundamental frequency of the spindle), the information contained in the Table 3.2 and the use of the Equation (3.16).

$$E = \frac{\bar{m}}{I} \left(\frac{\omega_n}{C_n} \right)^2 \tag{3.19}$$

3.4 Global FEA model

Allen [16] Presents a series of models for the calculation of the working volume of the FreeHex, wherein the working volume is calculated using parameters such as the position of the feet (bases for the linear actuators). These algorithms developed by Allen [16] can be used to extract the information required to feed a sub-program that generates an intermediary file to be read in to ANSYS [17], taking advantage of the parametric language used in ANSYS APDL. This code is able to establish the proper modes and frequencies within the working volume related to a configuration for the FreeHex.
Figure 3.7 shows the different components of the FreeHex and Table 3.3 explains the description of each component within the FEA model.

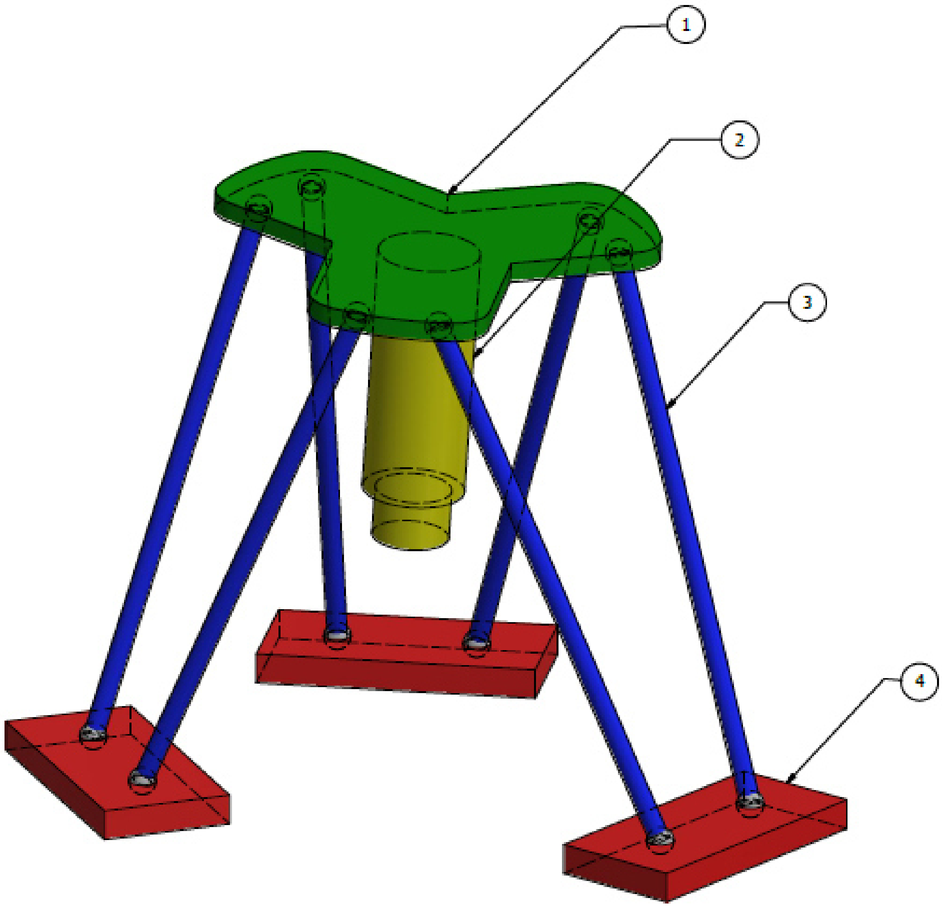

Figure 3.7. Components conforming the FreeHex

Component number	Component name	Type of element used
1	Upper platform	Shell
2	Spindle	Beam
3	Actuator	Link
4	Actuator Base	Solid

Table 3.3. Description of the components in the FreeHex.

As the program requires different coordinate systems, a principal Coordinate System (CS) is established in order to keep control of the mesh and the connections between components of the FreeHex. All the other joints of the FreeHex are referenced to this CS which also is used to solve the model. The process utilised for the acquisition of the proper modes and frequencies is described by the next diagram and the process is detailed in the next paragraphs.

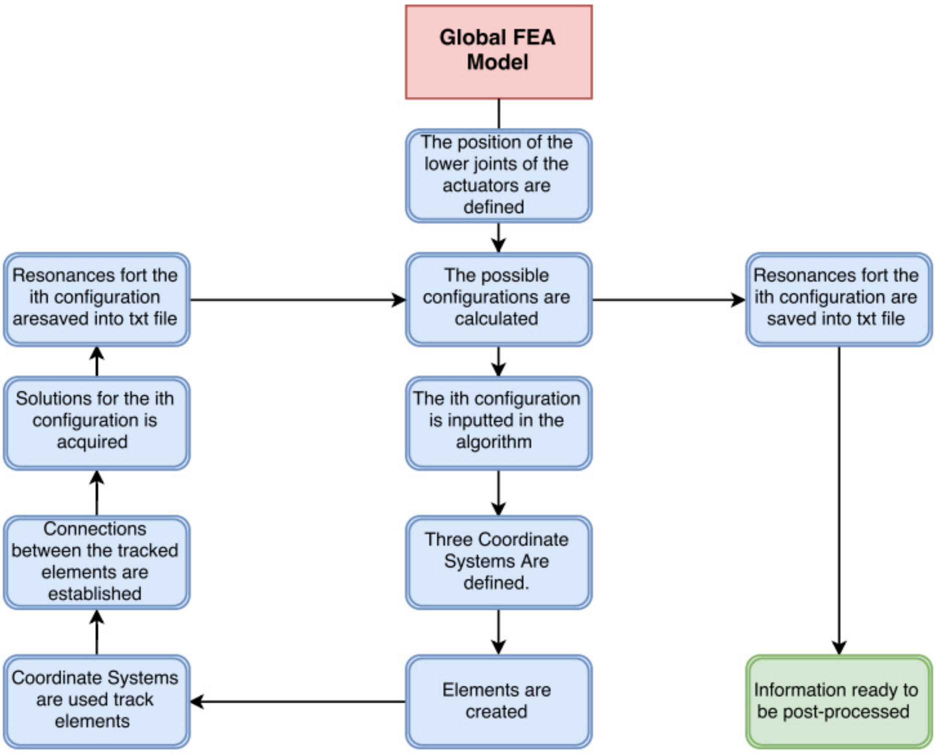

Figure 3.8. Diagram of the algorithms developed to estimate the resonances within an established working volume.

Three keypoints are defined between each pair of actuators with respect to the global coordinate system (CS_1) which compose each leg (Figure 3.10). These points are used to create and orientate the CS that will represent the coordinate origin for the FreeHex. Once this CS (CS_2) is declared the position of the upper joint is declared (referred to the machine CS_2). A third CS (called CS_3) is established with respect to the upper joint (in the sixth leg). This serves to create and mesh the upper platform. Later, the spindle holder and spindle are created and meshed. This process allows for easy tracking and relation of the connections (boundary conditions) between the different components created and improves the time and methodology of connection. These connections are modelled as the related to the original geometrical properties of the elements in association. Such connection is established through boundary constraint equations (rigid elements) between the nodes of the platform and the nodes of the spindle and actuator. As an example the node spindle laying closer to the same plane as the upper platform is connected with those of the platform with in the transverse radius of the spindle. As simplified connections would not result in the proper stiffness and stress distribution. The connections were compared by the analysis of the distribution of stress with respect to solid elements as well as the deformation generated by an arbitrary load as shown in Figure 3.9.

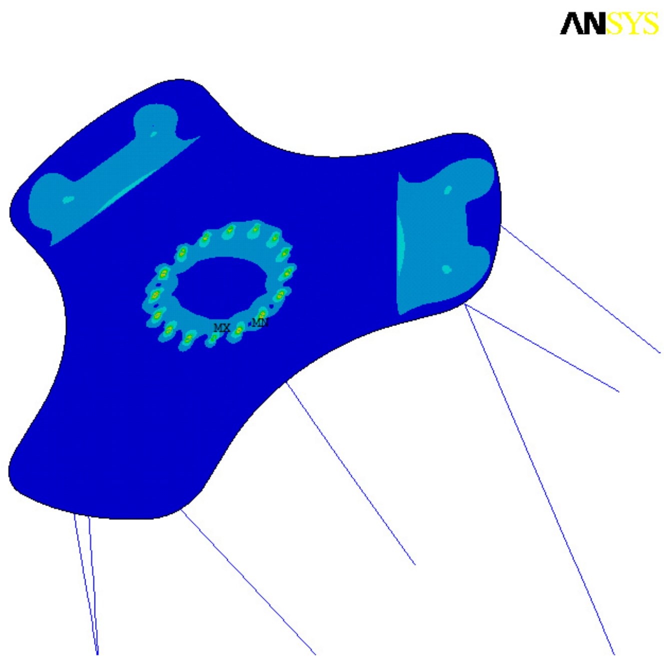

Figure 3.9. Stress distribution on the union between the spindle and the platform of the FreeHex.

The elements composing the linear actuators are then created and, given their boundary conditions in the system, declared to be link elements (articulated/articulated and such conditions leads to negligible bending effects). The conditions between the platform and the links are established by a series of boundary constraint equations, which are easily applied to the traceability of the related nodes, as well as the conditions for the lower joints.

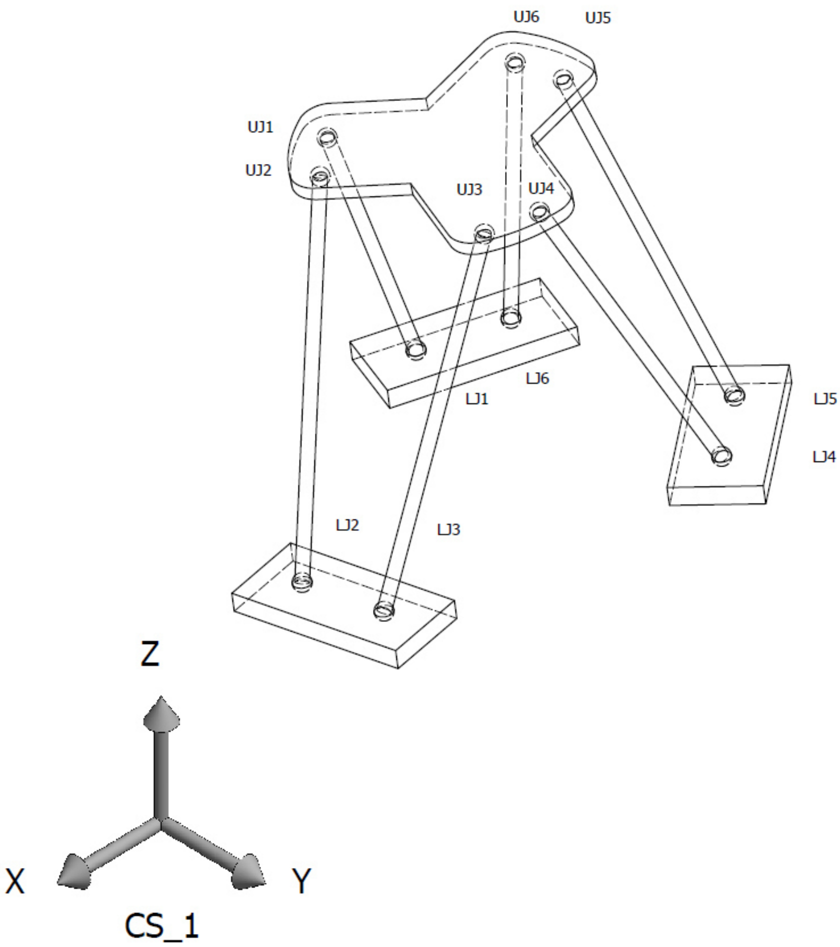

Figure 3.10. Diagram of the First Global Coordinate System (CS_1) used as a reference for the position of the base of the linear actuators.

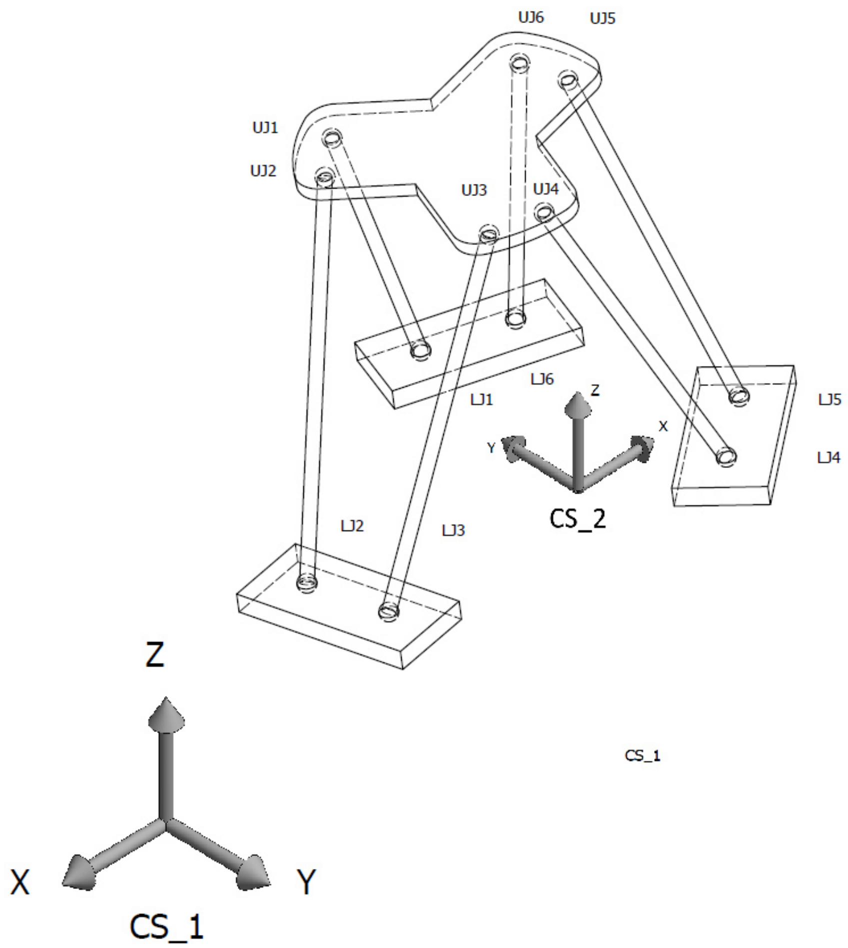

Figure 3.11. Diagram of the Machine Coordinate System (CS_2) used to evaluate the position of each joint with respect to an standard location.

The results obtained by this algorithm are then output into a second intermediary file for further analysis. The final interpretation is a resonance related to each position of the tool in the work envelope as this information can be used to avoid the undesired oscillatory effects in the machining operations.

3.5 Summary

In this chapter the process to acquire the resonate frequencies has been described. Beginning by the analysis of stiffness for the actuators in the operational range. From these results a function is established to relate range (or configuration) against stiffness of the actuator, in order to properly represent it by a link element. The mass and stiffness of the spindle are also calculated. Finally all the elements conforming the FreeHex are incorporated in a global model and related to the working volume, which is de terminated by the configuration of the pods.

Chapter 4

Experimental Analysis

This chapter presents the experimental methods followed to analyse the dynamical behaviour of the FreeHex. The structure is virtually divided into different sections to acquire its vibrational response to an excitation through impact testing, and the data acquired is then post-processed.

4.1 Introduction

The modal parameters such as resonance, damping and mode shapes allow the development of models that describe the behaviour of a given system. These modal parameters can be acquired from the Frequency Response Function (FRF), being measured in one or more positions in a structure. Resonances and damping values can be acquired in any position on the structure except the places that belong to a nodal position (where the nodal location is equivalent to zero). In order to acquire data with a high level of precision, it is preponderant to effectuate the measurements over a wide number of positions in order to evaluate all the structure under analysis. The FRF is acquired by measuring the signal over different channels and to be acquired, it is necessary to excite the structure. When the excitation is performed with a impact hammer, the usual procedure is to keep the accelerometers in one position and perform the excitation through the structure with a modal hammer. The estimation of the FRF with multiple inputs and multiple outputs (MIMO) is given Equation (4.1) and is illustrated in Figure 4.1.

$$[Y] = [H]\,[X] \tag{4.1}$$

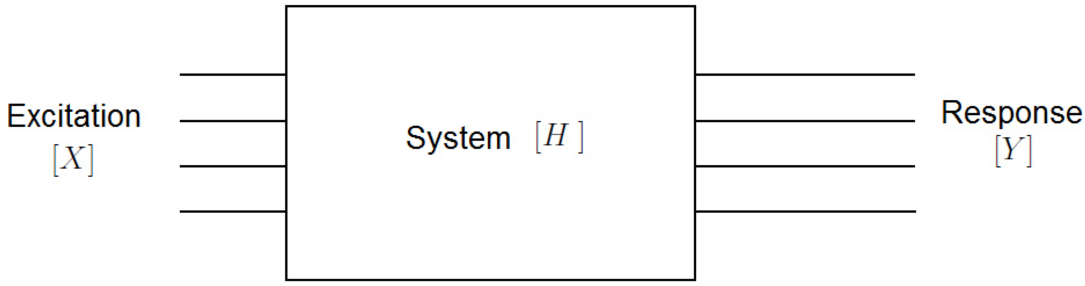

Figure 4.1. Multiple input multiple output schematics.

The expanded representation of Equation (4.1) is:

$$\begin{bmatrix} Y_1 \\ Y_2 \\ Y_3 \\ \vdots \\ Y_n \end{bmatrix} = \begin{bmatrix} H_{1,1} & H_{1,2} & H_{1,3} & \dots & H_{1,n} \\ H_{2,1} & H_{2,2} & H_{2,3} & \dots & H_{2,n} \\ H_{3,1} & H_{3,2} & H_{3,3} & \dots & H_{3,n} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ H_{n,1} & k_{n,2} & H_{n,3} & \dots & H_{n,n} \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ \vdots \\ X_n \end{bmatrix} \tag{4.2}$$

In many cases, the measurement of one column or one row of the transfer function $[H]$ contains sufficient information to extract the modal parameters of the structure under analysis. The assumption is that the selected position contains information of all the modes of vibration as the response is the sum of each individual response.

4.2 Configuration

The experimental analysis begun by placing the FreeHex over one of the fixtures used to perform machining operations for aerospace applications (Figure 4.2). This fixture was selected as it could be used to place the FreeHex in the same position without any major complications, ensuring repeatability over different sets of experiments. It was also ensured that the assembly of Fixture and FreeHex was isolated from the bench where it laid in order to avoid the contamination of the result from the base itself. This was made by the use of rubber isolators to later excite the assembly (FreeHex and Fixture) and measure the response of the bench, and vice versa.

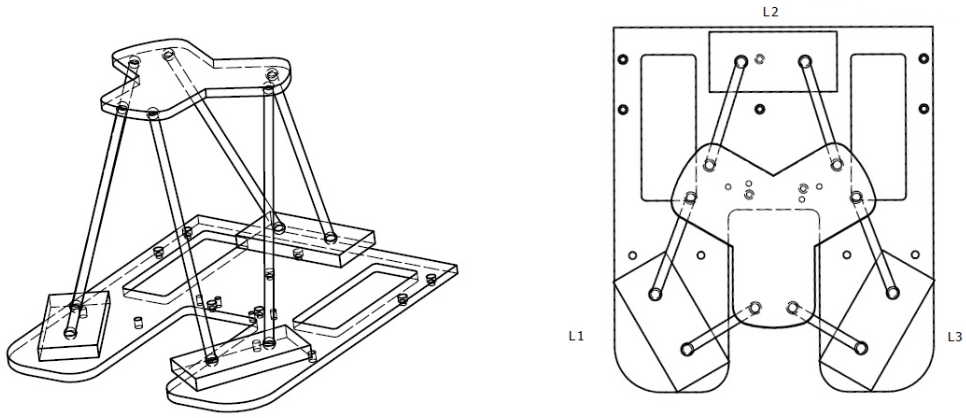

Figure 4.2. Placement of the FreeHex over a fixture for aerospace applications. L_n represent the pair of actuators 1, 2 and 3.

Once the mounting and isolation was completed, the next step involved the virtual segmentation of the structure in order to place and identify the nodes where the excitation and response of the structure would be measured. This procedure comprised equally spaced segments of the upper base of the FreeHex and the spindle (determinate by the use of the FEM, as the response could be more significant in these sections). Figure 4.3, shows the location of the nodes on the upper platform.

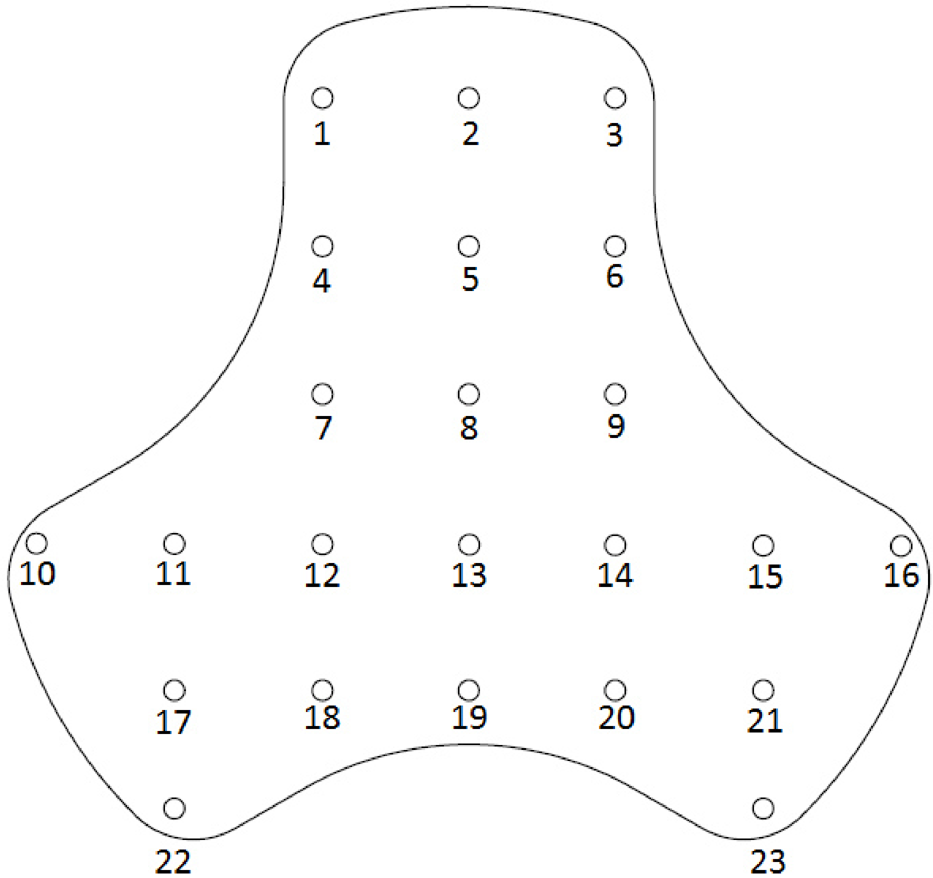

Figure 4.3. Representation of the nodes on the platform of the FreeHex, for the measurement of the response and excitation over the structure.

This method of segmentation was also applied to the spindle and spindle holder in order to acquire the resonances acting on the system components. Once the location of the nodes has been established, the acquisition system was conformed as Figure 4.4 shows.

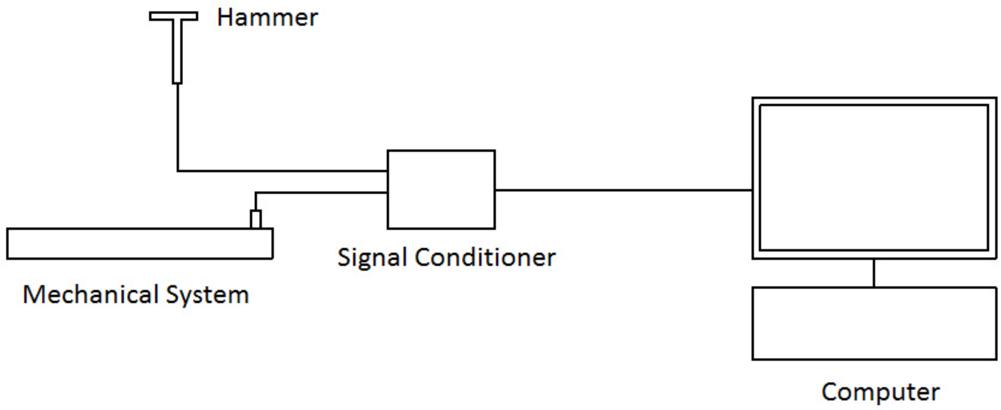

Figure 4.4. Representation of the acquisition system.

4.3 Acquisition

Once the system of acquisition was configured, the structure was excited on the different nodes marked in Figure 4.3, the accelerometer used remained in the same position, until all the nodes had been taped by impact hammer. Then the accelerometer was moved to the next node until all the nodes where excited. This process has been applied systematically as described to obtain the response of the system $[H]$, Figure 4.5.

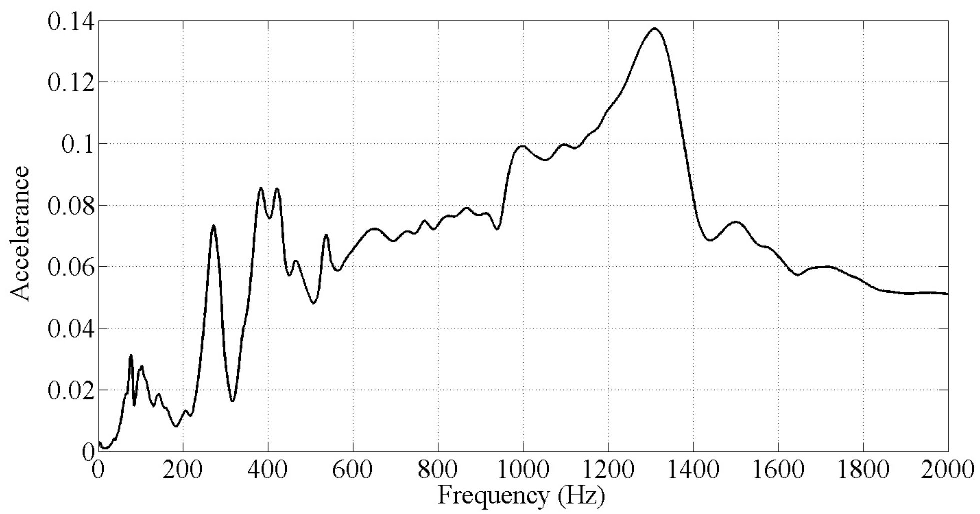

Figure 4.5. Frequency Response Function (output acceleration/input force) for the vertical modes of vibration of the FreeHex. This FRF has been obtained directly from the commercial software used to acquire the signals.

Frequency Response Function for the vertical modes of vibration of the FreeHex. This FRF has been obtained directly from the commercial software used to acquire the signals. The Figure 4.5 shows the FRF of the FreeHex for the vertical modes, this information has been extracted by the direct Fourier analysis of the components from the acceleration registered by the accelerometers and the signal acquired from the hammer. It is possible to see in the Figure 4.5 that in some ranges of the frequency the signal does not show single components that would correspond to the resonance values, but instead the response seems to reach a constant value for some frequency values. This information lead to perform a more advanced analysis of the data acquired.

4.4 Post Processing

Suppose a non stationary signal f_{ns} conformed by n sinusoid components with one signal acting over all the sampling time, and the rest of the signals present only at small periods of the sampling time. If f_{ns} is decomposed by methods as Fast Fourier Transform (FFT) the result shows an erroneous result. Mainly the error would show components of non existing frequencies in the spectrum of the signal. However more advanced methods have been developed such as the Welch Method described by Rahi [26] as a powerful process to reduce noise in a signal. This last tool is based on the definition of smaller windows to analyse signals, it acquires the components through FFT and averages the values of the amplitudes within the specified range [20].

The Welch Method has been applied in order to acquire the FRF of the FreeHex. Using a segment length of 3500 samples, overlapping such samples by 50 percent. This method allowed to reduce the noise in the acquired signal. Such perturbations in the signal are produced by secondary impacts of the internal components of the machine, given its complexity and the tolerances of the parts conforming the FreeHex. The FRF are displayed from Figure 4.6 thru 4.8. These three FRFs represent the response of the system in the vertical, horizontal an rotational directions as the intention is to capture all the possible modes of vibration in all the possible degrees of freedom. Is important to mention that the observed peaks in each plot represent zones of instability which correspond to the resonances of the system.

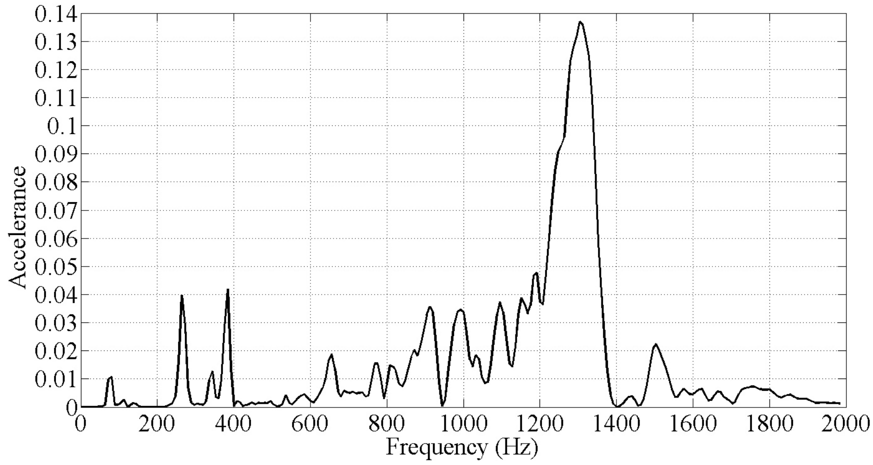

Figure 4.6. FRF of the FreeHex using the Welch Method. This FRF corresponds to the vertical modes of vibration of the FreeHex.

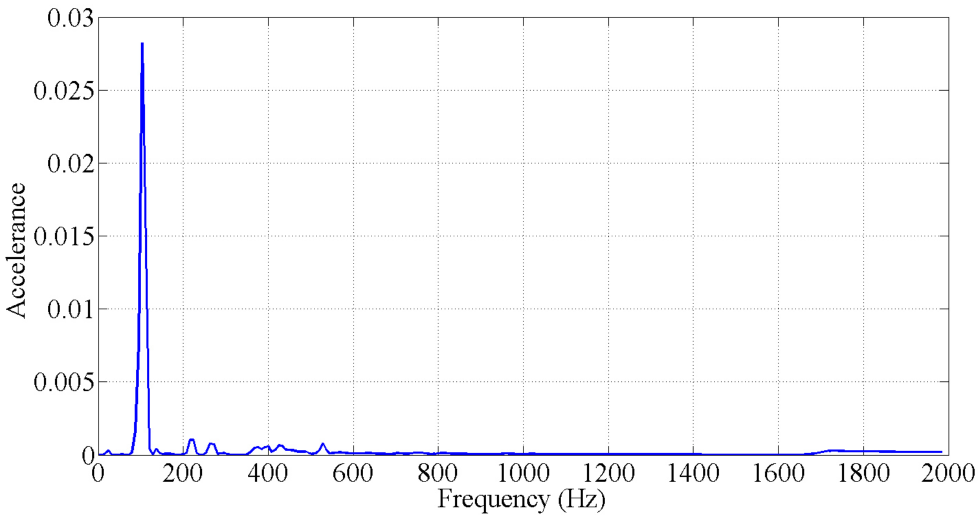

Figure 4.7. FRF of the FreeHex using the Welch Method. This FRF corresponds to the horizontal modes of vibration of the FreeHex.

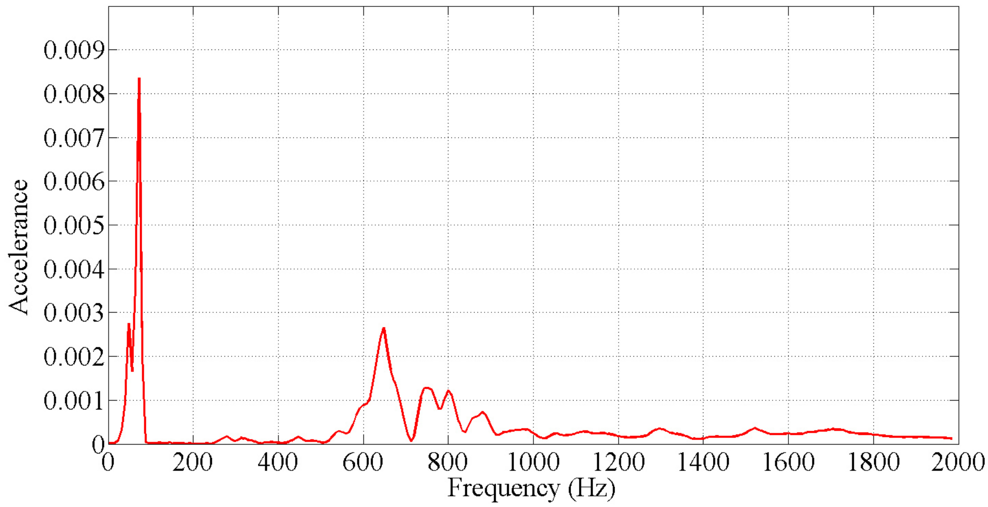

Figure 4.8. FRF of the FreeHex using the Welch Method. This FRF corresponds to the rotational modes of vibration of the FreeHex.

As the range of interest for the analysis is being considered between 0 Hz and 600 Hz the proceed information is limited to these values for the frequency. The resonances are easily distinguishable from the comparison in the Figure 4.9 as the pick values for the amplitude represent the proper values for the vertical, horizontal and rotational modes of vibration. Each peak is then selected and represented in Table 4.1.

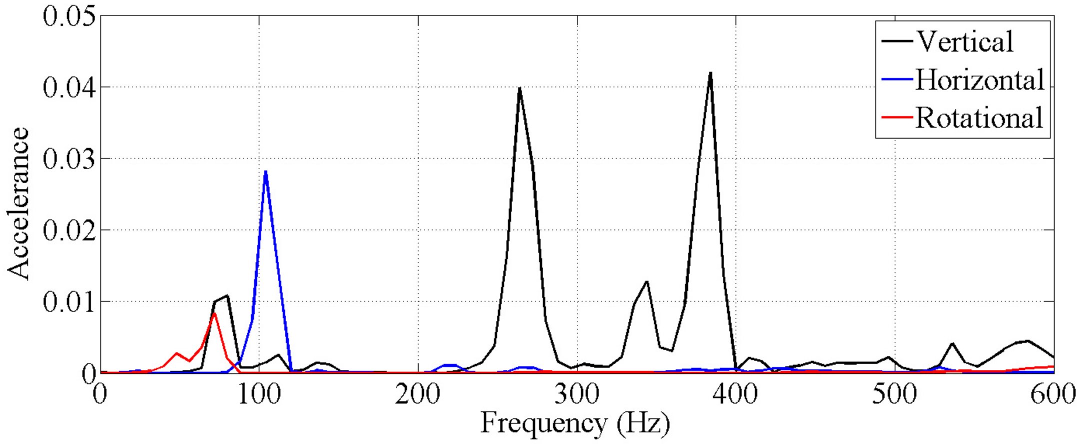

Figure 4.9. FRF (Welch Method) modes of vibration from 0 Hz to 600 Hz.

i	Frequency (Hz)
1	48
2	55
3	72
4	104
5	220
6	264
7	272
8	304
9	344
10	384
11	408
12	448
13	536
14	584

Table 4.1. Resonances acquired thru experimental analysis length.

4.5 Estimation of the Dissipation of Energy For The FreeHex

As the structure under analysis is a system composed of infinite degrees of freedom its response will include n modes and n resonances. Then if each mode and its correspondent resonance is isolated from the rest the method shown in section §2.4 can be applied to estimate the lost of energy in each mode. A rapid method to isolate each mode is thru the application of passband filters to the displacement signal (after the double integration of the acceleration). Finally the method explained in Section §4.4 can be easily applied.
As the estimation from two consecutive amplitudes for a cycle may result in an arbitrary selection for the complete signal an average of cycles is considered as the best approach. Then the average damping ratio would result in:

$$\zeta_{Avg} = \frac{1}{n}\sum_{i=1}^{n}\frac{\Delta_i}{\sqrt{4\pi^2 + \Delta_i^2}} \tag{4.3}$$

The damping ratios of each resonance are displayed on table 4.2.

Frequency (Hz)	Damping Ratio
48	0.035
55	0.045
72	0.046
104	0.032
220	0.034
264	0.037
272	0.029
304	0.033
344	0.031
384	0.024
408	0.030
448	0.026
536	0.030
584	0.039

Table 4.2. Damping ratios acquired through experimental analysis.

The values of the damping ratios are displayed in Figure 4.10. From Figure 4.10 is possible to see a pattern in the damping ratio values as the frequency varies. Several methods are tested and the behaviour is found to be of a quadratic function of the frequency expressed as:

$$\zeta(F) = 0.0456 - (8.55x10^{-05})F + 1.14x10^{-07}F^2 \tag{4.4}$$

Equation (4.4) is evaluated from 0 to 622 Hz and displayed over the values of the damping ratios in Figure 4.10. Equation (4.4) accurately represent the damping ratios as a function of frequency given that the correlation coefficient is equivalent to 0.71 ($0.7 < r \leq 1$ indicates strong correlation).

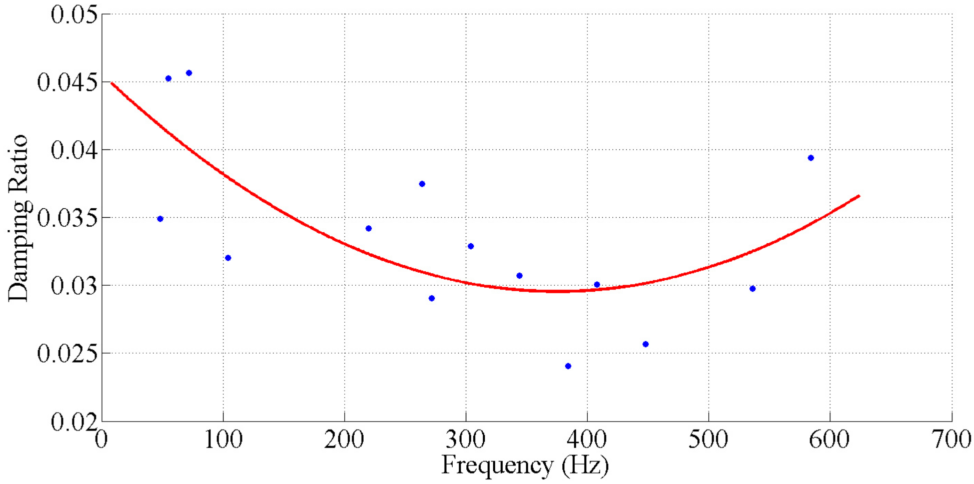

Figure 4.10. Damping ratios as function of frequency.

The Figure 4.10 represent the Rayleigh Damping of the structure which states that viscous damping is a linear combination of mass and stiffness. Rayleigh damping considers that mass proportional damping is dominant in lower frequencies and the stiffness proportional damping is dominant at higher frequencies. Then Equation (4.4) can be used in further studies of the structure in order to study is behaviour under more complex analysis.

4.6 Summary

In this chapter the methods followed to acquire the resonant frequencies have been explained. The FreeHex has been virtually divided into different sections where the structure has been exited to acquire the response. The signals acquired from the accelerometer have been processed using the Welch method in order to identify the resonant frequencies. Finally the damping coefficients have been calculated from the processed data, so these maybe used in future investigations.

Chapter 5

Comparison of Results and Conclusion

In this chapter the results of the Finite Element and experimental methods are presented and compared, from these comparisons conclusions of the project are presented as well as future work.

5.1 Introduction

The present document has shown the methodology used to obtain the most important vibrational characteristics of a Parallel Kinematic Machine. The elements used for this are described as:

- Finite element analysis was used to obtain in advance the resonances of the PKM. This method is proposed to relate each postilion of the FreeHex to a range of resonances. This methodology implies the analysis of each component to later be included in a global system of simplified geometry but with equivalent stiffness and mass distribution.

- The experimental section allows for the validation of the FEA models proposed. And could permit to use such model in to an infinite number of configurations for the FreeHex allowing to avoid conditions that could lead the machine to operate out of stable parameters.

5.2 Results Comparisons

Due to the fact that idealisations and simplifications performed on the FEA models could affect the predictions of the stiffness and mass behaviour of the represented structure a comparison between FEA model and experimental results shows to be of significant importance.

The range of interest has been established between 0 Hz and 600 Hz as the upper limit frequency represents a 150 % of the maximum rotational speed achieved by the spindle used in the FreeHex. So the comparison presented in Table 5.1 limits experimental and FEA results up to 600 Hz.

FEA (Hz)	Experimental (Hz)	Error (%)
47.60	48	0.83
55.65	55	1.19
62.98	72	12.52
125.82	114	5.26
217.26	220	1.25
264.75	264	0.28
288.89	272	6.21
310.15	304	2.02
364.04	344	5.83
370.49	384	3.52
434.83	408	6.58
440.29	448	1.72
531.36	536	0.87
550.56	584	5.73

Table 5.1. Comparison between experimental and FEA resonant frequencies.

The average error between the FEA models and experimental results lies in a value of 3.94 % showing a good correlation between both methods. The maximum error is on the third natural frequency with a value of 12.5 %. This significant difference is mainly do to the effects occurring in the joints of the actuators and the upper platform of the FreeHex. These effects represent very complex conditions and the inclusion of them would result in the opposite focus of this document.

As this significant difference between results is only present in one frequency the FEA is considered to be validated so it may be used in different configurations not only for a single position but for a complete working volume. This last sentence establishes that the spindle maybe controlled so it avoids rotational speeds that could bring the FreeHex to unstable operational conditions, being these conditions the so called resonances.

The already validated model is expanded to calculate the proper values of vibration on a symmetric configuration of the FreeHex for a complete working volume. Very basic information is imputed in to the algorithms that provide the code to analyse the structure as the coordinates of the centre of each actuator base. Later these codes are imputed into a commercial FEA software and the proper values corresponding to each coordinate are saved into a txt file, to be stored to generate the images shown in Figure 5.1.

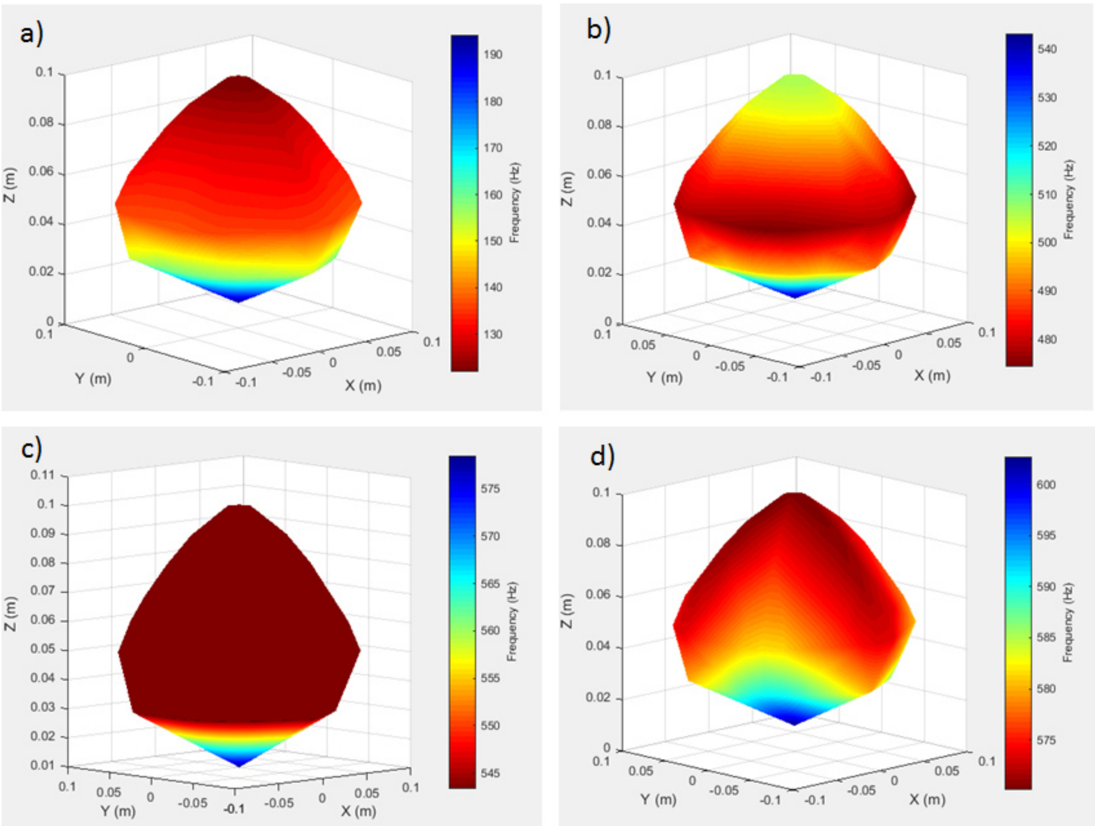

Figure 5.1. Representation of the working volume and the association of each position to the a)1st, b) 2nd, c) 3rd and 4th resonances. The colours displayed relate each value of the frequency of the resonances to the value displayed in the bar next to each plot.

The results displayed in Figure 5.1 do not consider the FreeHex laying over any base, but analyse each base of each actuator being restricted in all its degrees of freedom. It is also important to note that the system in both situations considers different mass distribution and stiffness properties. The model where the resonances are displayed for the complete working volume represents a simplified model and is able to study the characteristics of the FreeHex apart from any other subsystem. As the model has been evaluated and validated over more complex situations (over a base designed to place the FreeHex for this purpose) the model is still relieve.

5.3 Conclusions

- By the FEM all the components conforming the FreeHex have been accurately represented. Mass and stiffness properties represent the most important properties of a dynamical system.
- Systems conformed by different pieces can analysed by a deep study of each of its parts to latter joint its individual results to accurately represent the system.
- By the analysis of the operational length of each actuator a function has been established to describe the stiffness of each actuator. Complicated elements can be represented in their most basic form thorough the appropriated simplifications as boundary conditions and loads.
- The addition of any support or external element to the FreeHex will result in a significant change of the response of the global system as properties such as stiffness and mass are modified.

5.4 Observations

- A simplified representation of a complex system will result in a reduction of time and resources to represent its structural behaviour.
- When a signal changes through time several windows can be applied to track the change in its harmonics.
- The application of methods such as Welch eliminate noise but can also reduce the resolution of the signal. This loss of information can be compensated by the proper treatment of the data acquired.
- The best method to obtain damping properties is trough experimentation. Then experimentation can significantly improve predictions given by FEA. Resulting in a loop of improvement.
- Methods as the decay of the amplitude between to maximum values of displacement resulted to be useful to accurately represent the lost of energy of the system.

5.5 Future Work

The development of highly efficient algorithms can be continued and the methods here presented shall be considered in such studies. These algorithms shall be robust enough to be implemented in parallel computing. Such algorithms will be applied to understand and improve the behaviour of PKM during different operations. These algorithms must include the forces developed during cutting operations. Such conditions would impose loads over the tool and the contact between the base of the linear actuators. Simulation of such forces could significantly improve the applications of Parallel Kinematic Machines [27]. These algorithms shall also consider the forces generated during the movement of the FreeHex combined with the structural deformations that will result. This last statement could be used to improve the final result where PKM can be applied. As a deeper understanding of the excitations for PKM could be gained and therefore smoother operations can be achieved.

These algorithms could also be used during the design process of new PKMs, working together with neural networks in order to create a self improvement process. Such techniques would lead to a deeper understanding of the design prior to its creation. Even adaptive designs to operation could be created by the use of these algorithms and the inclusion of new advanced materials. These materials could be able to modify its behaviour as electrical impulses controlled by the algorithms would actuate them in the process of machining. Even the modification of containers for fluid could be implemented to change the distribution of mass to modify, as desired, the dynamical behaviour of the new PKMs.

Even a standard configuration could be established to evaluate the requirements of maintenance for the PKM, as the FRF would be changing as the components start to wear. The prediction of the ideal FRF could be monitored and established by the algorithms developed.

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